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OUTSTANDING SCHOLARSHIP EXEMPLAR



Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Scholarship 2024 Physics

Time allowed: Three hours
Total score: 32

Check that the National Student Number (NSN) on your admission slip is the same as the number at the top of this page.

You should answer ALL the questions in this booklet.

For all 'describe' or 'explain' questions, the answers should be written or drawn clearly with all logic fully explained.

For all numerical answers, full working must be shown and the answer must be rounded to the correct number of significant figures and given with the correct SI unit.

Formulae you may find useful are given on page 3.

If you need more room for any answer, use the extra space provided at the back of this booklet.

Check that this booklet has pages 2–24 in the correct order and that none of these pages is blank.

Do not write in any cross-hatched area (⚡). This area will be cut off when the booklet is marked.

YOU MUST HAND THIS BOOKLET TO THE SUPERVISOR AT THE END OF THE EXAMINATION.

The formulae below may be of use to you.

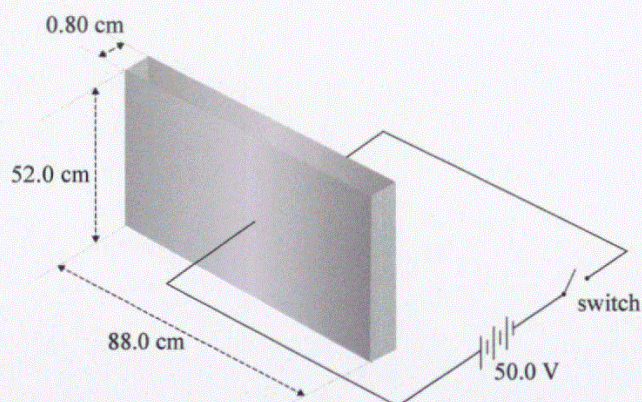
$v_f = v_i + at$ $d = v_i t + \frac{1}{2} at^2$ $d = \frac{v_i + v_f}{2} t$ $v_f^2 = v_i^2 + 2ad$ $F_g = \frac{GMm}{r^2}$ $F_c = \frac{mv^2}{r}$ $\Delta p = F \Delta t$ $\omega = 2\pi f$ $d = r\theta$ $v = r\omega$ $a = r\alpha$ $W = Fd$ $F_{\text{net}} = ma$ $p = mv$ $x_{\text{COM}} = \frac{m_1 x_1 + m_2 x_2}{m_1 + m_2}$ $\omega = \frac{\Delta\theta}{\Delta t}$ $\alpha = \frac{\Delta\omega}{\Delta t}$ $L = I\omega$ $L = mvr$ $\tau = I\alpha$ $\tau = Fr$ $E_{K(\text{ROT})} = \frac{1}{2} I\omega^2$ $E_{K(\text{LIN})} = \frac{1}{2} mv^2$ $\Delta E_p = mg\Delta h$ $\omega_f = \omega_i + \alpha t$ $\omega_f^2 = \omega_i^2 + 2\alpha\theta$ $\theta = \frac{(\omega_i + \omega_f)}{2} t$ $\theta = \omega_i t + \frac{1}{2} \alpha t^2$	$T = 2\pi\sqrt{\frac{l}{g}}$ $T = 2\pi\sqrt{\frac{m}{k}}$ $E_p = \frac{1}{2} ky^2$ $F = -ky$ $a = -\omega^2 y$ $y = A \sin \omega t$ $y = A \cos \omega t$ $v = A\omega \cos \omega t$ $v = -A\omega \sin \omega t$ $a = -A\omega^2 \sin \omega t$ $a = -A\omega^2 \cos \omega t$ $\Delta E = Vq$ $P = VI$ $V = Ed$ $Q = CV$ $C_T = C_1 + C_2$ $\frac{1}{C_T} = \frac{1}{C_1} + \frac{1}{C_2}$ $E = \frac{1}{2} QV$ $C = \frac{\epsilon_0 \epsilon_r A}{d}$ $\tau = RC$ $\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2}$ $R_T = R_1 + R_2$ $V = IR$ $F = BIL$ $V = BvL$ $F = Bqv$ $F = Eq$ $E = \frac{V}{d}$	$\phi = BA$ $\epsilon = -\frac{\Delta\phi}{\Delta t}$ $\epsilon = -L \frac{\Delta I}{\Delta t}$ $\frac{N_p}{N_s} = \frac{V_p}{V_s}$ $E = \frac{1}{2} LI^2$ $\tau = \frac{L}{R}$ $I = I_{\text{MAX}} \sin \omega t$ $V = V_{\text{MAX}} \sin \omega t$ $I_{\text{MAX}} = \sqrt{2} I_{\text{rms}}$ $V_{\text{MAX}} = \sqrt{2} V_{\text{rms}}$ $X_C = \frac{1}{\omega C}$ $X_L = \omega L$ $V = IZ$ $f_0 = \frac{1}{2\pi\sqrt{LC}}$ $v = f\lambda$ $f = \frac{1}{T}$ $n\lambda = \frac{dx}{L}$ $n\lambda = d \sin \theta$ $f' = f \frac{V_w}{V_w \pm V_s}$ $E = hf$ $hf = \phi + E_K$ $E = \Delta mc^2$ $\frac{1}{\lambda} = R \left(\frac{1}{S^2} - \frac{1}{L^2} \right)$ $E_n = -\frac{hcR}{n^2}$
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QUESTION ONE: CAPACITOR TANK

Permittivity of free space = $8.85 \times 10^{-12} \text{ F m}^{-1}$

Relative permittivity of air = 1.0

Hannah uses a rectangular tank, as shown in the diagram, to carry out a series of experiments investigating the behaviour of a parallel plate capacitor. The front and back of the tank are made of conducting metal plates 88.0 cm wide and 52.0 cm high. The sides and bottom of the tank are 0.80 cm wide, and are made of thin, non-conducting plastic.



- (a) (i) For her first experiment, Hannah closes the switch to connect the plates to the 50.0 V power supply.

Show that the charge stored on the plates is $2.5 \times 10^{-8} \text{ C}$.

Explain why this value should be stated to two significant figures.

$$\text{Capacitance of the tank} = C = \frac{\epsilon_0 \epsilon_r A}{d}$$

$$C = \frac{(1)(8.85 \times 10^{-12})(0.88 \times 0.52)}{(0.008)}$$

$$C = 5.06 \times 10^{-10} \text{ F}$$

$$Q = CV \Rightarrow Q = (5.06 \times 10^{-10})(50)$$

$$Q = 2.53 \times 10^{-8} \text{ C} \approx 2.5 \times 10^{-8} \text{ C}$$

The given values are at 2 s.f., thus the accuracy required is only to 2 s.f., and at a magnitude of 10^{-8} , the extra 0.03×10^{-8} is relatively insignificant.

- (ii) Explain why the amount of energy stored in the charged capacitor is only half of the total energy provided by the power supply.

$$E_c = \frac{1}{2} QV = \frac{1}{2} (2.5 \times 10^{-8})(50) = \cancel{6.33} 6.3 \times 10^{-7} \text{ J}$$

$$E_p = QV = (2.5 \times 10^{-8} \text{ C})(50 \text{ V}) = 1.3 \times 10^{-6} \text{ J}$$

Half the energy lost is due to energy dissipating as heat through the wires, which have their own resistance.

- (b) The tank can be filled with a non-conducting oil with a relative permittivity of 2.1. No charge is transferred from the plates to the oil. There is a valve in the bottom of the tank that can be opened to drain the oil from the tank.

For her second experiment, Hannah starts with the tank empty, the plates uncharged, and the switch open. She then carries out the following steps in sequence to achieve a potential difference between the plates that is greater than 50.0 V:

- Step 1: Close the switch.
 Step 2: Fill the tank with oil.
 Step 3: Open the switch.
 Step 4: Drain the oil from the tank.

- (i) Explain the effect of each of these steps, and how this sequence results in a potential difference that is greater than 50.0 V between the plates.

Step 1: the switch closes, allows the "empty" capacitor to charge up to 50V and $2.5 \times 10^{-8} \text{ C}$, as per usual.

Step 2: the tank is filled with oil, with relative permittivity of 2.1. The capacitance increases by 2.1 times, but since the switch is closed, the potential difference remains 50V and thus the quantity of charge increases by 2.1 times ($Q = CV$).

Step 3: the switch is opened, breaking the circuit. No current (and thus charge) flows or moves from the capacitor plates.

Step 4: the oil is drained, decreasing the capacitance by 2.1 times. Because charge is conserved, the potential difference (originally ^{still} 50V) increases by 2.1 times (more than 50V) due to $Q = CV$.

- (ii) Calculate the value of the potential difference between the plates that results from following the sequence of steps described on page 5.

Step 1: $Q_1 = 2.5 \times 10^{-8} \text{ C}$, $C_1 = \frac{5.1 \times 10^{-10} \text{ F}}{1.1}$, $V_1 = 50 \text{ V}$.

Step 2: $C_2 = \frac{5.1}{1.1} \times 10^{-10} \times 2.1 = 1.1 \times 10^{-9} \text{ F}$.

V_2 stays 50 V , $Q_2 = C_2 V_2 \Rightarrow Q_2 = (1.1 \times 10^{-9})(50)$

Original values: $Q_1 = 2.5 \times 10^{-8} \text{ C}$, $C_1 = 5.06 \times 10^{-10} \text{ F}$, $V_1 = 50 \text{ V}$.

Step 2: $C_2 = C_1 \times 2.1$, $Q_2 = C_2 V_2 = 2.1 \times C_1 \times V_1 = 2.1 \times Q_1$

since $V_2 = V_1 = 50 \text{ V}$ while switch closed.

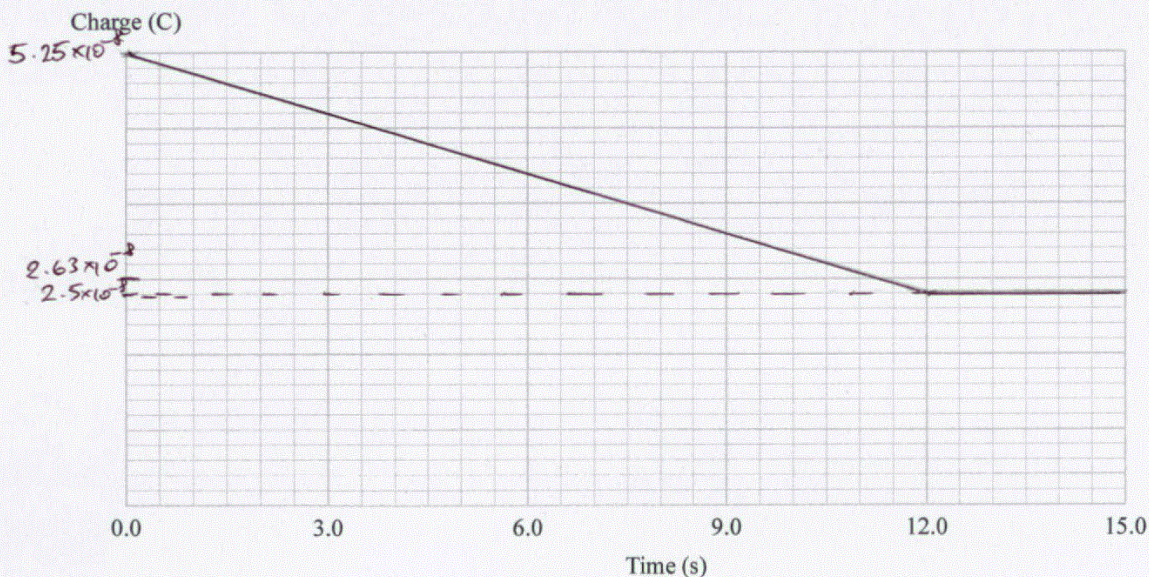
Step 3/4: $Q_3 = Q_2 = 2.1 \times Q_1$ since charge conserved by closed switch.

→ extra paper on back, to page(19)

- (c) For her third experiment, Hannah fills the tank with oil and closes the switch so that the potential difference across the plates is 50.0 V . At time $t = 0.0 \text{ s}$, she opens the valve and oil drains from the tank while the switch remains closed. The tank is completely empty after 12.0 s . Assume the oil flows from the tank at a constant rate.

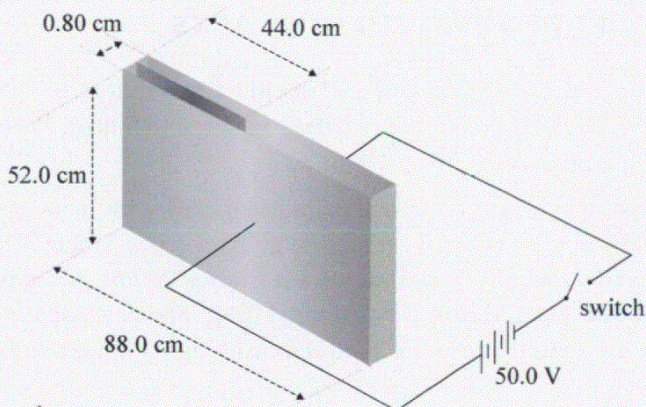
Sketch a graph on the axes below to show how the charge stored on the plates changes over the first 15.0 s .

Indicate key values on the charge axis.



- (d) For her last experiment, Hannah now adds a third parallel thin metal plate between the two original plates. The new plate is the same height as the two original plates, but is only half the width, and has negligible thickness.

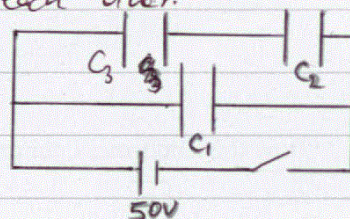
Using appropriate physics concepts, explain the effect that adding the third plate will have on the total capacitance of the tank.



The metal plate is not a dielectric as it is a conductor. Because it only has half the width, it effectively makes three capacitors - one in parallel to two capacitors in series with each other.

Setup is shown on the right.

Because C_1 has only half the area A of the original capacitance C_0 it has half the capacitance, $\frac{1}{2}C_0$. C_3 and C_2 have half the area A , but also half the separation d . Using $C = \frac{\epsilon_0 \epsilon_r A}{d}$, the effects cancel out and C_3 and C_2 have the same capacitance as the original, C_0 .



Total capacitance of C_3 and C_2 in series: $\frac{1}{C_{3+2}} = \frac{1}{C_3} + \frac{1}{C_2} = \frac{1}{C_0} + \frac{1}{C_0}$.
 $C_{3+2} = \frac{1}{2}C_0$

Total capacitance of the tank (C_{3+2} and C_1 in parallel):

$$C_T = C_{3+2} + C_1 = \frac{1}{2}C_0 + \frac{1}{2}C_0 = C_0$$

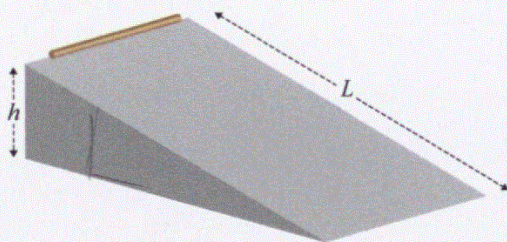
$C_T = C_0$. No effect on total capacitance

No change, still the same.

QUESTION TWO: MARTIAL ARTS

Several cultures around the world have a history of martial arts using a staff. Some examples include Japanese Bōjutsu, Portuguese Jogo Do Pau, and Māori Mau Rākau.

Ben is practising using a cylindrical staff with mass m and radius r . Ben places the staff on its side at the top of a slope with length L and vertical height h , and releases it from rest. The cylindrical staff rolls freely down the slope without sliding. The rotational inertia I of a solid cylinder about a central axis along its length is $I = \frac{1}{2}mr^2$.



Source: www.festival.nz/article/a-window-into-te-ao-m%C4%81ori-the-taiaha/

- (a) Show that the time, t , taken for the staff to reach the bottom of the slope is given by:

$$t = L\sqrt{\frac{3}{gh}}$$

where g = the acceleration due to gravity.

Energy before: $E_p = mgh$, all converted after L meters to

kinetic and rotational energy $\Rightarrow \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{v}{r}\right)^2$$

$$gh = \frac{1}{2}v^2 + \frac{1}{4}v^2 = \frac{3}{4}v^2$$

$$gh = \frac{3}{4}\left(\frac{L}{t}\right)^2 = \frac{3L^2}{4t^2} \Rightarrow t^2 = \frac{3L^2}{4gh} \Rightarrow t = \frac{L}{2}\sqrt{\frac{3}{gh}}$$

v is the velocity at the bottom of the slope. Due to constant acceleration, this is equal to twice the average velocity.

$$\text{Average velocity} = \frac{L}{t} \cdot \left(\frac{dv}{dt}\right), \quad v = \frac{L}{t}$$

$$v = 2\left(\frac{L}{t}\right)$$

$$v = \left(\frac{L}{t}\right)\left(\frac{1}{2}\right)$$

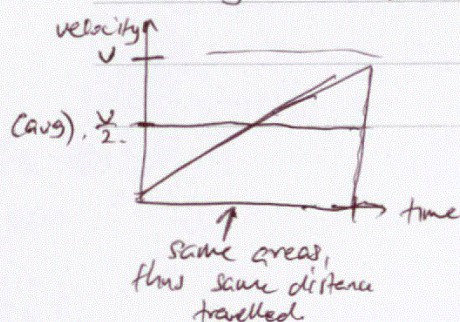
$$gh = \frac{3}{4}v^2 = \frac{3}{4}\left(\frac{L}{t}\right)^2 = \frac{3}{4}\left(\frac{L^2}{t^2}\right)$$

$$gh = \frac{3}{4}v^2 = \frac{3}{4}\left(\frac{2L}{t}\right)^2 = \frac{3}{4}\left(\frac{4L^2}{t^2}\right)$$

$$gh = \frac{3L^2}{t^2}$$

$$t^2 = \frac{3L^2}{gh}$$

$$t = L\sqrt{\frac{3}{gh}}, \text{ as required.}$$



(b) By referring to relevant physics principles, discuss the validity of these two statements:

- (i) **Statement 1:** For an object with fixed mass to have linear acceleration, it must experience a non-zero net force.

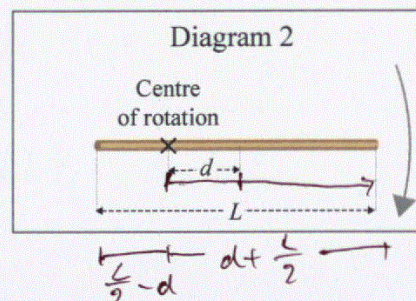
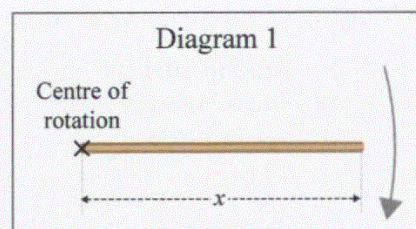
$\Sigma F = ma$, Newton's second law. The sum of forces (net force) must have a resultant acceleration on an object with mass. for $a \neq 0$, $\Sigma F \neq 0$, thus the net force must ~~not~~^{not} be 0 and thus will be ~~a~~ unbalanced to have linear/constant acceleration. Thus, Statement 1 is valid.

- (ii) **Statement 2:** For an object with fixed mass to have angular acceleration, it must experience a non-zero net torque.

$\Sigma \tau = I\alpha$. The net torque has a proportional effect on angular acceleration, α , by rotational inertia I . If the net torque $\Sigma \tau = 0$, then $\alpha = 0$ for any I . Thus, for $\alpha \neq 0$, ~~$\Sigma \tau \neq 0$~~ ^{$\Sigma \tau \neq 0$} , and a non-zero^{net} torque is unbalanced, thus there must be an unbalanced, non-zero net torque to have angular acceleration. Thus statement 2 is valid.

- (c) For a uniform thin rod with total length x and mass m rotating about one end, as shown in Diagram 1, the rotational inertia I is given by $I = \frac{1}{3}mx^2$.

While performing a sideways strike, a staff can be modelled as a thin, uniform rod rotating about a vertical axis through the centre of rotation, which is in a fixed position. The centre of rotation is closer to one end of the rod than the other, as shown in Diagram 2. The rotational inertia of a thin, uniform rod rotating about an axis part way along the rod can be found by adding the rotational inertias of the sections of the rod either side of the centre of rotation.



Show that the rotational inertia of a thin, uniform rod of length L and mass m rotating about a point a distance d from the centre of mass is given by:

$$I = m \left(\frac{L^2}{12} + d^2 \right)$$

since uniform mass distributed proportionally.

Short side has rotational inertia $I_1 = \frac{1}{3} \left(\frac{L}{2} - d \right)^2 \left(\frac{\frac{L}{2} - d}{L} \right) (m)$

$$I_1 = \frac{1}{3} \left(\frac{m}{L} \right) \left(\frac{L}{2} - d \right)^3$$

Long side $I_2 = \frac{1}{3} \left(\frac{L}{2} + d \right)^2 \left(\frac{\frac{L}{2} + d}{L} \right) (m)$

$$I_2 = \frac{1}{3} \left(\frac{m}{L} \right) \left(\frac{L}{2} + d \right)^3$$

$$I = I_1 + I_2 = \frac{1}{3} \left(\frac{m}{L} \right) \left(\frac{L}{2} - d \right)^3 + \frac{1}{3} \left(\frac{m}{L} \right) \left(\frac{L}{2} + d \right)^3$$

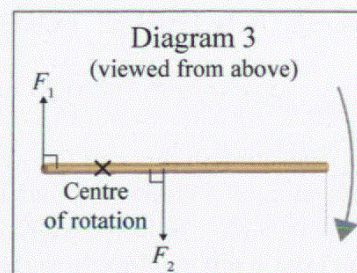
$$I = \frac{1}{3} \left(\frac{m}{L} \right) \left[\left(\frac{L}{2} - d \right)^3 + \left(\frac{L}{2} + d \right)^3 \right]$$

$$I = \frac{1}{3} \left(\frac{m}{L} \right) \left[\frac{L^3}{8} + \frac{3L^2}{4}(-d) + \frac{3L}{2}(d^2) + (-d)^3 + \frac{L^3}{8} + \frac{3L^2}{4}(d) + \frac{3L}{2}(d^2) + (d^3) \right]$$

$$I = \frac{1}{3} \left(\frac{m}{L} \right) \left[\frac{L^3}{4} + 3Ld^2 \right] = m \left(\frac{L^2}{12} + \frac{3d^2}{3} \right)$$

$$I = m \left(\frac{L^2}{12} + d^2 \right) \text{ as required.}$$

- (d) Ben makes the staff rotate around a vertical axis by applying two horizontal forces F_1 and F_2 , one with each hand. These forces push the ends of the staff in opposite directions, as shown in Diagram 3 (viewed from above). The forces F_1 and F_2 produce a net torque that gives the staff an angular acceleration about the centre of rotation. The forces F_1 and F_2 are spaced equally from the centre of rotation and are applied perpendicular to the staff's length. The centre of rotation stays in the same position.



- (i) Discuss the relative sizes of the forces F_1 and F_2 needed to cause the staff to have an angular acceleration while the centre of rotation does not move.

~~The forces are required to be of the same size or magnitude due to them being equidistant from the center of rotation.~~

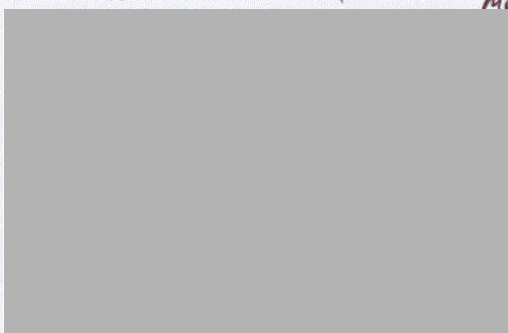
The forces cannot be the same size - F_2 must be greater. This is to compensate for the ~~at~~ higher rotational inertia of the longer side of the staff, so that the torques are ~~not~~ unbalanced and the torque of F_2 is greater than that of F_1 .

Thus $\Sigma \tau \neq 0$ and there is angular acceleration. The center of mass may move but this is not the center of rotation, which does not move.

- (ii) A sideways strike can be blocked by holding a second staff vertically in front of the rotating staff.

Explain the position and direction that the blocking force should be applied to stop the rotating staff most effectively.

Explain the effect this blocking force has on the motion of the rotating staff.



Source: www.reddit.com/r/martialarts/comments/13cdmpy/mau_rakau_and_wharetutaua/?rdt=43199

When the staffs connect, a force is applied against the rotating staff. This is torque that opposes the rotation of the staff, and the blocking staff or force should be perpendicular to the staff's ^{rotating} motion upon contact and as close to the end of the staff. This is where the linear velocity v is greatest, thus the impulse and force applied is greatest, and the distance from the pivoting point is greatest. (or). Since F and r are maximized, the torque $\tau = Fr$ is maximized and most effectively blocks the rotating staff.

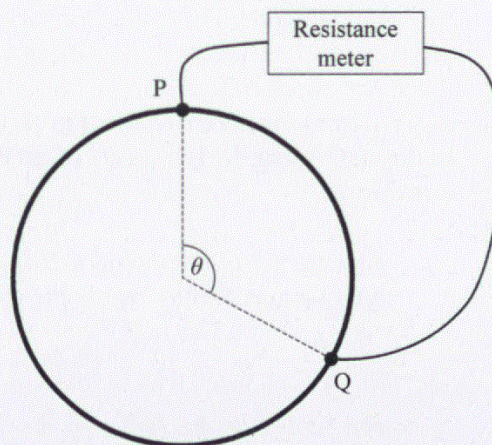
QUESTION THREE: LOOPS AND RINGS

Acceleration due to gravity = 9.81 m s^{-2}

A long, straight piece of thin resistive wire has a total resistance R between one end and the other. The piece of wire is now bent into a circular shape and the ends are connected together to make a single circular loop.

A resistance meter is connected between two points on the loop, P and Q, which are separated by an angle θ around the loop as shown.

Point P remains fixed at the top of the loop, but point Q can be moved to any point around the edge of the loop.



- (a) By considering the resistance of the two sections of the circular loop (clockwise from P to Q, and anticlockwise from P to Q), show that R_{PQ} , the total resistance of the loop between the points P and Q, is given by the expression:

$$R_{PQ} = \frac{R\theta}{2\pi} \left(1 - \frac{\theta}{2\pi} \right)$$

$$\text{Resistance anticlockwise} = R \left(1 - \frac{\theta}{2\pi} \right) \quad R \left(\frac{2\pi - \theta}{2\pi} \right) = R \left(1 - \frac{\theta}{2\pi} \right)$$

$$\text{Resistance clockwise} = R \left(\frac{\theta}{2\pi} \right)$$

Effectively this is a parallel circuit with two resistors.

$$\frac{1}{R_T} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{1}{R \left(1 - \frac{\theta}{2\pi} \right)} + \frac{1}{R \left(\frac{\theta}{2\pi} \right)}$$

$$\frac{1}{R_{PQ}} = \frac{1}{R} \left(\frac{\frac{\theta}{2\pi}}{\frac{\theta}{2\pi} \left(1 - \frac{\theta}{2\pi} \right)} \right) = \frac{1}{R} \left(\frac{1}{1 - \frac{\theta}{2\pi}} \right)$$

$$R_{PQ} = R \left(\frac{\theta}{2\pi} \left(1 - \frac{\theta}{2\pi} \right) \right)$$

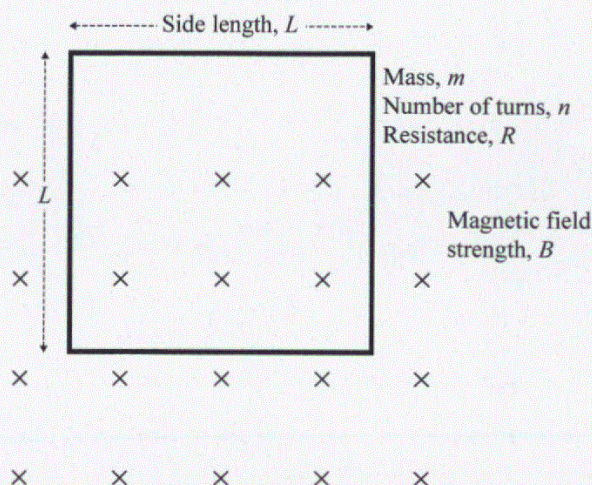
$$R_{PQ} = \frac{R\theta}{2\pi} \left(1 - \frac{\theta}{2\pi} \right) \quad \text{as required}$$

- (b) Explain, using physics concepts, where point Q should be placed to give the largest value of R_{PQ} , and determine the maximum value of R_{PQ} for a length of wire with resistance $R = 10 \Omega$.

$R_{PQ} = R \left(\frac{\theta}{2\pi} - \frac{\theta^2}{4\pi^2} \right)$ Maximum resistance when the two resistances are equal, i.e. $\theta = \pi$. This is because there will be equally resistive paths, rather than one less resistive path (which more charge/current flows through) and one more resistive path, which has less overall resistance. Thus Q should be placed directly opposite P, so $\theta = \pi$.

$$R_{PQ} = \frac{10\pi}{2\pi} \left(1 - \frac{\pi}{2\pi} \right) = \underline{\underline{2.5 \Omega}}$$

- (c) A square coil of wire with mass, m , resistance, R , and number of turns, n , falls freely into a uniform magnetic field as shown. The effects of air resistance can be ignored.



- (i) By considering the forces acting on the coil, show that the terminal velocity of the coil as it falls into the field is given by:

$$v = \frac{mgR}{B^2 L^2 n^2}$$

Terminal velocity $\Rightarrow$ constant velocity, forces are balanced.

$$F_g = mg, \quad F_B = nBIL, \quad U = nBL \cdot I = IR$$

$$F_g = F_B \Rightarrow mg = nBIL \Rightarrow mg = nBL \left(\frac{V}{R} \right)$$

$$mg = nBL \left(\frac{nBLv}{R} \right) \Rightarrow mg = \frac{n^2 B^2 L^2}{R} v$$

$$v = \frac{mgR}{B^2 L^2 n^2} \text{ as required}$$

$\mathcal{P}$ (note. $F_B = BIL$ for one loop, so for n loops/coils
 $F_B = nBIL$. Same for $U = nBL \Rightarrow U = nBL$)

- (ii) By choosing realistic values for m , n , and L , estimate the terminal velocity of a small coil with resistance $R = 1.0 \, \Omega$ as it falls into a magnetic field of strength $B = 1.0 \times 10^{-3} \, \text{T}$.

Discuss the motion of your coil as it enters the field.

$$v = \frac{mgR}{B^2 L^2 n^2} \quad \text{mass } m \approx 100 \text{ g} \approx 0.1 \text{ kg.}$$

$$n \approx 100 \text{ loops/coils/turns.} \quad L = 15 \text{ cm} = 0.15 \text{ m.}$$

$$v = \frac{(0.1)(9.8)(1)}{(1 \times 10^{-3})^2 (0.15)^2 (100)^2} = 4360 \text{ ms}^{-1}.$$

The coil is initially accelerating due to gravity. As it enters the field, the magnetic force slows/decreases the acceleration, so while the coil still continues falling faster, it does so at a slower rate. Eventually, the forces are balanced and the net acceleration is 0, and the small coil reaches terminal, constant velocity; as shown above.

- (d) A square coil is unwound, and the same piece of wire is used to make a second square coil with sides twice as long as the original coil.

Use physics concepts to explain how the terminal velocity of the new coil compares to that of the initial coil when dropped into the same magnetic field.

Resistance is the same, mass is the same. Sides twice as long ($2L$) ~~force~~ will decrease the terminal velocity in the same magnetic field to the same g by $\frac{1}{4}$. However, for the same piece of wire (same total length), this means the number of coils must have halved ($\frac{N}{2}$). This ~~fact~~ decreases the force and thus influence of the magnetic field by $\frac{1}{4}$ on the coil, thus the terminal velocity increases by 4 times. The change in length's impact on the force of the magnetic field is exactly counteracted by the change in the number of coils because fundamentally, it is an identical wire moving through the field. Thus, there is no difference in the terminal velocities.

QUESTION FOUR: LASER COOLING

Planck's constant = $6.63 \times 10^{-34} \text{ J s}$

Mass of a sodium atom = $3.82 \times 10^{-26} \text{ kg}$

Speed of light = $3.00 \times 10^8 \text{ m s}^{-1}$

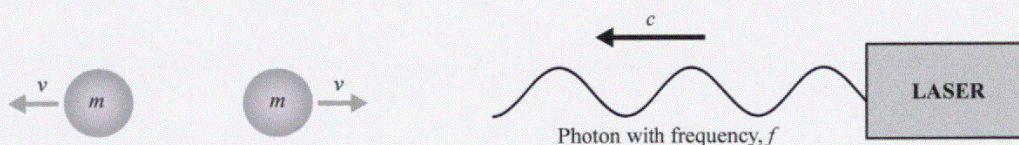
Physicists use lasers to transfer momentum and slow down gaseous atoms. This can cool a gas down to temperatures close to absolute zero.

The momentum of a photon is related to its wavelength by the relationship:

$$p = \frac{h}{\lambda}$$

A laser of frequency, f , is shone into a container holding gaseous sodium atoms. The energy of each photon is slightly less than the difference in energy between two electron energy levels in a sodium atom.

Gaseous sodium atoms can be moving in any direction, but for the purposes of Question Four, we will only consider one-dimensional motion – either towards or away from the laser.



- (a) Explain which sodium atom, the one moving towards the laser or the one moving away from the laser, can absorb a photon from the laser.

Towards. The sodium atom moving towards the laser experiences a compression of the laser wavefronts, thus the observed light undergoes Doppler shift and the frequency f increases. Because the energy of each photon is $E = hf$ (proportional to frequency), the higher frequency photon has more energy, and thus is able to fill the energy difference between two electron energy levels in the sodium atom, thus electrons are able to move to an excited state and the sodium atom can absorb it. The atom moving away, observes a lower frequency and energy photon which cannot fill the energy difference, and thus cannot absorb the photon.

- (b) Explain how the momentum of each of the sodium atoms, the one moving towards the laser and the one moving away from the laser, may be affected by the photon.

The momentum of the atom moving toward the laser will decrease. Because the photon and atom are moving in opposite directions towards each other, total momentum is their difference in momentum. As a result, the magnitude of the momentum of the sodium atoms moving toward the laser would likely decrease due to the photon. The atom moving away from the laser will ~~increase~~ and the photon are both travelling in the same direction, thus their total momentum is the sum of their individual momenta. As a result, it is likely that the atom's momentum will increase due to the photon, if it collides or potentially absorbs (if the photon is of higher frequency).

- (c) For an observer moving at speed v , where v is much less than the speed of light, the apparent change in frequency Δf of an electromagnetic wave of frequency f , due to the Doppler effect, can be approximated by:

$$\Delta f = \frac{fv}{c}$$

A transition between energy levels in a sodium atom occurs when a photon with wavelength $5.89 \times 10^{-7} \text{ m}$ is absorbed. The average speed of a gaseous sodium atom at room temperature is $5.70 \times 10^2 \text{ m s}^{-1}$.

Calculate the percentage change in momentum of a sodium atom caused by the absorption of a photon with wavelength $5.89 \times 10^{-7} \text{ m}$.

$$c = \lambda f \Rightarrow f = \frac{3 \times 10^8}{5.89 \times 10^{-7}} = 5.09 \times 10^{14} \text{ Hz}$$

$$\Delta f = f_0 - f = f \frac{v}{c} = 5.09 \times 10^{14} \left(\frac{5.7 \times 10^2}{3 \times 10^8} \right) = 9.68 \times 10^8 \text{ Hz}$$

$$\Delta E = h \Delta f = (6.63 \times 10^{-34}) (9.68 \times 10^8) = 6.42 \times 10^{-25} \text{ J}$$

$$\text{original energy} = \frac{1}{2} m v^2 = \frac{1}{2} (3.82 \times 10^{-26}) (5.7 \times 10^2)^2 = 6.21 \times 10^{-21} \text{ J}$$

$$\text{new velocity} \Rightarrow 6.21 \times 10^{-21} + 6.42 \times 10^{-25} = \frac{1}{2} (3.82 \times 10^{-26}) v^2$$

$$v = 570.03 \text{ m s}^{-1}$$

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Question Four continues
on the following page.

- (d) Despite being used for slowing atoms down, this process can sometimes result in an atom moving faster than it was originally. After absorbing a photon, the sodium atom can then emit a photon when the electron drops back down to its original energy level. The emitted photon can be emitted in any direction.

Explain how this process can result in the atom moving faster than its original speed if the photon is emitted in the same direction the original photon was travelling (away from the laser).

Conservation of momentum requires the initial and final ^{total} momentum of the system to be equal. When the photon is emitted in the same direction as the original ~~laser~~ ^{photons} (ie to the left) this momentum is counteracted by ~~mean~~ an increase in the momentum of the sodium atom in the opposite direction, towards the laser (in the same direction as it was travelling originally). The photon momentum and the increase in sodium momentum are in opposite directions and thus cancel, leaving total momentum conserved, but as a result, the sodium atom moves faster in the same direction after emitting the photon.

Extra space if required.
Write the question number(s) if applicable.

QUESTION
NUMBER

Q1 b) ii) from page 6 $\rightarrow$.

Removing oil decreases capacitance by 2.1 times $\rightarrow C_3 = \frac{1}{2.1} C_2$
 $= \frac{A\epsilon}{d} \frac{1}{2.1} \times 2.1 \times C_1 = C_1$.

$$Q_3 = C_3 V_3 \Rightarrow 2.1 \times Q_1 = C_1 \times V_3 \Rightarrow V_3 = 2.1 \times \frac{Q_1}{C_1}$$

$$\text{since } Q_1 = C_1 V_1 \Rightarrow V_1 = \frac{Q_1}{C_1}$$

$$V_3 = 2.1 \times V_1, \quad V_1 = 50V.$$

$$V_3 = 2.1 \times (50)$$

$$V_3 = 105V.$$

Q4 c) $\rightarrow$ from page 17.

$$P_0 = \text{original momentum of atom} = mv = (570) \times (3.82 \times 10^{-26})$$

$$= \underline{2.18 \times 10^{-23} \text{ kg ms}^{-1}}$$

$$\text{photon frequency } f = \frac{c}{\lambda} = \frac{3 \times 10^8}{5.89 \times 10^{-7}} = 5.09 \times 10^{14} \text{ Hz.}$$

$$\text{observed frequency} = f_0 = f + \Delta f.$$

$$\Delta f = f \frac{v}{c} = 5.09 \times 10^{14} \left(\frac{570}{3 \times 10^8} \right) = 9.68 \times 10^8 \text{ Hz.}$$

$$f_0 = 5.093 \times 10^{14} \text{ Hz.} \Rightarrow \lambda_0 = \frac{c}{f} = \frac{3 \times 10^8}{5.093 \times 10^{14}} = 5.889 \times 10^{-7} \text{ m.}$$

$$\text{momentum } p = \frac{h}{\lambda_0} = \frac{6.63 \times 10^{-34}}{5.889 \times 10^{-7}} = 1.126 \times 10^{-27} \text{ kg ms}^{-1}.$$

$$P_{\text{TOT}} = 2.18 \times 10^{-23} + 1.126 \times 10^{-27} = 2.178 \times 10^{-23} \text{ kg ms}^{-1}.$$

After absorption, only atom left.

$$P_{\text{TOT}} = mv \Rightarrow P_{\text{TOT}} \text{ is the momentum of the sodium atom after.}$$

$$\% \text{ change} = 100 \times \frac{P_{\text{TOT}} - P_0}{P_0} = \frac{2.178 \times 10^{-23} - (570)(3.82 \times 10^{-26})}{(570)(3.82 \times 10^{-26})} \times 100$$

$$= 0.0052\% \text{ increase.}$$

Outstanding Scholarship

Subject: Physics

Standard: 93103

Total score: 28

Q	Score	Marker commentary
1	7	High level scholarship understanding of a novel capacitance context.
2	7	High level scholarship understanding of mechanics in the angular context given. Clear evidence of mathematical rigour.
3	8	Outstanding understanding of a variety of electrical and mechanics concepts. High level mathematical manipulation demonstrated.
4	6	Scholarship level understanding of modern physics in a unique and challenging context.